Solutions to Church & Ware

Chapter 4

Market Power and Dominant Firms

2. (a) Since
$$S(q) = \frac{f}{cq} + 1 > 1$$
there exist global economies of scale. This also means that $AC(q) > MC(q) = c$ for all output levels.

   (b) If the incumbent can commit to charge a price equal to marginal cost, then there will not be entry. An entrant would not be able to cover its fixed costs and would earn negative profits. This result does not depend on whether f is sunk or not. Either way, the entrant will not be able to recover f if they produce.

4. (a) From Exercise 4.2, the profits from selling the durable are \$450,000. The per period profits from selling a nondurable as a function of its marginal cost are
$$\pi = \frac{(1000 - c)^2}{4}.$$
Since there are two periods and the discount factor is 1, the non-durable is more profitable if
$$2\frac{(1000 - c)^2}{4} > 450,000.$$
This is true if $c \leq 51.32$.

   (b) From Exercise 4.2, the profits from leasing the durable are \$500,000, which are always greater than the profits from selling the nondurable provided its marginal cost is positive.

   (c) The efficient solution is to produce 1000 units of the durable product, since it is costless and at that level of output willingness to pay equals zero—marginal cost. The monopolist reduces durability—engages in planned obsolescence—to avoid the Coase Conjecture.

6. (a) For $P \geq p^0 = 20$, the supply function for the fringe is $Q^s = P$.

   (b) The residual demand function for the dominant firm is $Q^D(P) = 100 - 2P$.

   (c) $P^* = 26$.

   (d) Monopoly profits are 2401; the profits of the dominant firm 1152 Total surplus under monopoly is 3601.5, and the total surplus when there is a dominant firm is 4028. In this case a dominant firm is more efficient than a monopolist as it results in a larger total surplus.

Chapter 5

Non-Linear Pricing and Price Discrimination

1. The marginal price is $p = 0.20$. The monopolist is able to extract all of the surplus in this case, so $CS = 0$, and $PS = 0.32$.

3. (a) The uniform monopoly price is $p^m = 3$.

   (b) The price charged to the under 25 group is $p = 2.80$, and the price charged to the over 25 group is $p = 3.50$.

   (c) Each group should pay the marginal cost of each drink, plus a cover charge equal to their consumer's surplus at that point. Both types pay 2.00 per drink. Those under 25 pay a cover charge of $CS(p = 2.00) = 6.40$, and those over 25 pay a cover charge of $CS(p = 2.00) = 9.00$.

   (d) At a price of 2.00 per drink, students obtain a surplus of 6.40. Thus, this is the maximum cover charge that will still attract both types of consumers.

   (e) After midnight all consumers are students, so the monopolist should charge the optimal student price of $p = 2.80$. Before midnight the optimal price is 3.15. The monopolist is practicing partial third degree price discrimination. The profits for part (d) are \$6.40 and the profits for part (e) are \$3.78.

5. (a) The manufacturer should charge Type 1 individuals \$4.00, their valuation for the good. Since their time cost is \$1.25, the coupon would have to offer at least this large a discount before the Type 1's are tempted to use them.

   (b) At a price of \$4.00, Type 2 individuals will only buy cereal if they receive a discount of \$1.00, the difference between their valuation and the price that they must pay.

   (c) Using parts (a) and (b), the manufacturer should set a price of \$4.00 and have a coupon discount of \$1.00. This means that he is making a profit of \$1.50 from the Type 1's, and \$0.50 from the Type 2's. An increase in manufacturing costs by \$0.50 would reduce profits from the Type 2 individuals to zero, so there is no longer any incentive to offer coupons.

Chapter 6

Market Power and Product Quality

2. Firm 1 should choose the highest quantity that would not be profitable for a low quality firm at price v_H; i.e. q such that the low quality producer is indifferent between $(p = v_L, 10)$ and (v_H, q). Since the producer can only sell up to 10 units, his profit maximizing strategy is to restrict supply to 2.5 units and set a price of v_H.

4. Using the expression in the text, expected lifetime cost equals 200.

6. (a) The monopolist chooses $s^m = \frac{1}{3}$, $q^m = \frac{1}{3}$, and $p^m = \frac{2}{9}$.

   (b) Consumer's surplus evaluated at $q^m = \frac{1}{3}$ is $\frac{1}{18}s$. Profits evaluated at $q^m = \frac{1}{3}$ are $\frac{2}{9}s - \frac{1}{3}s^2$.

   (c) At $q^m = \frac{1}{3}$, the socially efficient level of quality is $s = \frac{5}{12}$. Thus, given the monopolist's choice of quantity, he chooses an inefficiently low level of quality.

Chapter 8

Classic Models of Oligopoly

2. (a) $q_1^C = q_2^C = q^C = 60$
   $\pi_1^C = \pi_2^C = \pi^C = 3600$
   $P^C = 80$

   (b) See Figure 8.1.

   (c) Because of symmetry and constant unit costs, the joint profit-maximizing price is the monopoly price. The profit-maximizing monopoly output sets its marginal revenue $(200 - 2Q)$ equal to marginal cost (20): $Q^m = 90$. The monopoly price is $P^M = 110$ and monopoly profits are $\pi^M = 8100$. Any combination of q_1 and q_2 that satisfies $q_1 + q_2 = 90$ implements the monopoly outcome. Letting the share of firm 1 be λ (where $0 < \lambda < 1$) then the profits of the two firms also sum to 8100 with $\pi_1 = \lambda 8100$ and $\pi_2 = (1 - \lambda)8100$. For instance if the two firms split the monopoly output equally $(\lambda = 1/2)$ then $q_1 = q_2 = 90$ and $\pi_1 = \pi_2 = 4050$.

   (d) The marginal revenue for firm *i* is $MR_i = 200 - q_j - 2q_i$. Substituting in $q_i = \lambda 90$ and $q_j = (1 - \lambda)90$, $MR_i = 110 - 90\lambda$ which is greater than marginal cost (20) for $0 < \lambda < 1$. For instance when they split the monopoly output equally, $\lambda = 1/2$ and $MR_i(45, 45) = 65 > MC_i = 20$. The optimal defection for each firm to produce on its best-response function. If $q_j = (1 - \lambda)90$ then

   $$q_i^D = \frac{180 - (1 - \lambda)90}{2}.$$

   If $\lambda = 1/2$, then $q_i^D = 67.5$. Since each firm has an incentive to increase output, the collusive agreement is unstable.

   (e) The free-entry number of firms is $N^C = 8$.

4. Using the formulas from the preceding question, in the premerger Cournot equilibrium: $Q^C = 55$, $P^C = 45$, $\pi_1^C = 400$, $\pi_2^C = 1225$, $CS^C = 1512.5$, and $TS^C = 3137.5$, where *CS* is consumer surplus and *TS* is total surplus. Postmerger the monopoly outcome is: $Q^M = 45$, $P^M = 55$, $\pi_1^M = 2025$, $CS^M = 1012.5$, and $TS^M = 3037.5$. On efficiency grounds the merger should be blocked since total surplus falls.

6. (a) If there is no trade then each firm acts as a monopolist in its own country. The equilibrium quantities are $Q^M = 18$ and $P^M = 22$.

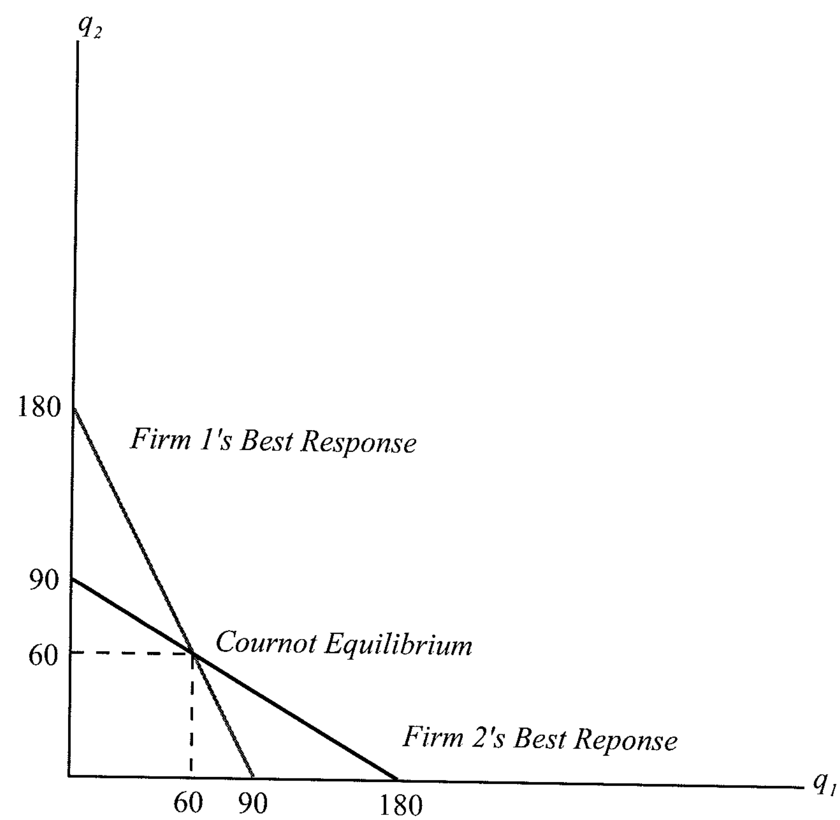

Figure 8.1: Problem 8.2

(b) KJ's marginal revenue on the first unit that they sell to the United States is equal to the monopoly price in the United States: $MR_{KJ}(0, 18) = P_{US}^{M} = 22$, the monopoly price since it does not have any inframarginal units.

(c) In the trade case there is Cournot duopoly in each country. The equilibrium in each country is: $q_i^C = 12$, $P^C = 16$, $\pi_i = 144$, and $CS^C = 288$. In the no trade case, $\pi_i^M = 324$ and $CS^M = 162$. Consumers prefer the trade equilibrium since consumer surplus is greater. While, the profits of each firm fall from 324 to 288 (144 in each market), total surplus increases with trade by 90 in each country. The trade equilibrium without arbitrage is an example of market segmentation or third degree price discrimination.

(d)
- Australia and U.S. with transportation costs of \$3: $c_{KJ} = 4$ and $c_{NW} = 7$. Using the results from Problem 3, the equilibrium in Australia is:
  $q_{KJ} = 13$, $q_{NW} = 10$, $P_A = 17$, $\pi_{KJ} = 169$, $\pi_{NW} = 100$, $CS_A = 264.5$ and $TS_A = 533.5$. The United States equilibrium is symmetric.
- Australia with \$3 per unit transportation cost and \$3 per unit tariff: $c_{KJ} = 4$ and $c_{NW} = 10$. Using the results from Problem 3, the equilibrium in Australia is:
  $q_{KJ} = 14$, $q_{NW} = 8$, $P_A = 18$, $\pi_{KJ} = 196$, $\pi_{NW} = 64$, $CS_A = 242$ and $TS_A = 562$.
  US with \$3 per unit transportation cost: $c_{NW} = 4$ and $c_{KJ} = 7$. The equilibrium in the U.S. is:
  $q_{NW} = 13$, $q_{KJ} = 10$, $P_{US} = 17$, $\pi_{NW} = 169$, $\pi_{KJ} = 100$, $CS_{US} = 264.5$ and $TS_{US} = 533.5$.
  With the tariff Australian consumers' welfare is reduced by 22.5. However, the profits of KJ increase (summing across the two countries) by 27 and the Australian government earns revenues of 24. Without retaliation the total gains from trade for Australia increase by 28.5.
- Australia and U.S. with a tariff of \$3 per unit transportation cost and \$3 per unit tariff: $c_{KJ} = 4$ and $c_{NW} = 10$. Using the results from Problem 3, the equilibrium in Australia is:
  $q_{KJ} = 14$, $q_{NW} = 8$, $P_A = 18$, $\pi_{KJ} = 196$, $\pi_{NW} = 64$, $CS_A = 242$ and $TS_A = 562$. The United States equilibrium is symmetric. The total gains from trade received by Australia equal 526, less than the 533.5 earned when there is no trade protection.

(e) Even with retaliation, the gains from trade for Australian's will increase by approximately 5 (4.89) by imposing a tariff of \$1 per unit. There is a trade off involved. Imposing a tariff raises the domestic price, decreasing consumers surplus and retaliation reduces profits in the export market. However, there is a resource saving (reduced transportation costs) and a gain in profits from switching to low-cost domestic producers and away from high-cost foreign producers.

8. Consider the incentives for firm 1 to charge a price higher than p^h. A price greater than p^h ensures that it will be the high price firm. In firm 1's case, given that it is going to be the high price firm and thus that $q_2 = k_2$, firm 1's profit-maximizing choice is to sell $q_1 = R_1(k_2)$ units which it can by charging p^h. Hence it does not want to charge a price greater than p^h. By construction, firm 1's profits are lower if it charges a price less than p^l. Recall the derivation of p^l.

   In order to see that firm 2 does not have an incentive to deviate, we need to derive firm 2's equilibrium profits. By construction, its expected profit from playing any price in the interval

$[p^l, p^h]$ is the same, otherwise it would not be willing to mix over prices in the interval. Its profits if it plays p^l will be $p^l k_2$ since it will be the low price firm and it is capacity constrained. Clearly, firm 2 also does not want to charge a price less than p^l. Doing so will continue to guarantee that it is the low price firm but since it is capacity constrained at p^l its profits will be reduced as its price falls. Moreover, firm 2 will not find it profitable to charge a price higher than p^h. Doing so will guarantee that it is the high priced firm. The best that firm 2 can do if it is the high-price firm is earn profits of $P(k_1 + R_2(k_1))R_2(k_1)$ which are its Cournot profits if firm 1 makes sales of k_1. However, $P(k_1 + R_2(k_1))$ is less than $P(R_1(k_2) + k_2)$ since $k_1 > k_2$. Firm 1 charges a higher price than firm 2 when it acts as a monopolist on its residual demand curve because its residual demand is larger than that of firm 2 since the capacity of firm 1 is larger. As a result, firm 2 will not find it profitable to charge a price greater than p^h.

Chapter 11

Product Differentiation

2. There is a unique equilibrium where firms 1 and 2 are paired at 1/6, firm 3 is located at 3/6, and firms 4 and 5 are paired at 5/6. The equilibrium market lengths of firms 1, 2, 4, and 5 is 1/6, while the equilibrium market length of firm 3 is 2/6.

4. There are two extremes when there are six firms:

   - (i) the distance between firms 3 and 4 is minimized and equal to 0. Firms 1 and 2 are paired at 1/6, firms 3 and 4 are paired at 3/6, and firms 5 and 6 are paired at 5/6.
   - (ii) the distance between firms 3 and 4 is maximized and equal to 1/4. Firms 1 and 2 are paired at 1/8, firm 3 is at 3/8, firm 4 locates at 5/8, and firms 5 and 6 are paired at 7/8.

   When firms 3 and 4 are paired all firms have the same market length. Call this length l. Then the two peripheral firms have a total market of $2l$. The distance between a paired interior firm and firm 3 or 4 must be $2l$ so that it (and firm 3 or 4, whichever is closest) has market length of l. In total this is $4l$. Solving $2l + 4l = 1$ for l gives $l = 1/6$, the market length of each firm.

   The upper bound is found by observing that the distance between firms 3 and 4 cannot exceed $2l$ or a firm would relocate between them. Hence the upper bound is found by setting $d = 2l$ and solving $2l + 4l + 2l = 1$ or $l = 1/8$.

   There is an equilibrium for any d such that $0 \leq d \leq 1/4$ where d is the distance between firms 3 and 4. We can use the formula $2l + 4l + d = 1$ to solve for l given any $0 \leq d \leq 1/4$.

6. (a) The Nash equilibrium for this location game is the set of locations such that firms do not have an incentive to relocate given the location of their rivals.

   (b) The two drinking establishments are both located half-way between the two towns.

   (c) In this case we would expect the drinks to taste the same. This also means that the market in this case does not produce any variety: product differentiation is minimized. Of course this result depends on government regulation that eliminates price competition, so it is not just the market that is responsible for the lack of variety.

   (d) No, the Nash equilibrium is not the socially optimal one. In the Nash equilibrium 2/3 of the population has to travel half the distance between the two towns. The socially optimal location would minimize aggregate travel costs by locating one bar in each town, in which case 2/3 of the population incurs 0 travel costs.

(e) The Nash equilibrium set of locations has firms 1 and 2 paired at 1/6 and firms 3 and 4 paired at 5/6. The market length for each firm is 1/4.

8. Since prices are fixed such that $(p-c) = 1$ and the number of consumers has been normalized to 1, density is 1 and profits equal market length. There are multiple equilibria: there can be either 4, 5, or 6 firms in equilibrium. With four firms the equilibrium locations are for firms 1 and 2 to be paired at 1/4 and firms 3 and 4 paired at 3/4. The profits for each firm are 1/12. If a fifth firm entered, the locations of the firms would be 1 and 2 paired at 1/6, firm 3 at 3/6, and firms 5 and 6 paired at 5/6. With entry of five firms, the profits of firms 1, 2, 4 and 5 are zero, but the middle firm (3) earns profits of 1/6. If six firms were to enter, then the symmetric equilibrium locations would be 1 and 2 paired at 1/6, 3 and 4 paired at 3/6, and 5 and 6 paired at 5/6. In this case the profits of all firms are zero.

   For an equilibrium number of firms there are also multiple equilibria. A firm's number does not imply an entry order, just a name. For example in the case of 6 firms, another equilibrium is for firms 5 and 6 to be paired at 1/6, 1 and 2 to be paired at 3/6, and 3 and 4 to be paired at 5/6.

10. A monopolist with a single product would serve the entire market, charging $p^m(1) = V - k$ and earning profits of $\pi^m = V - k - f$. A two product monopolist would set its prices to make the consumer at 1/2 indifferent between buying either product or its outside option (utility of 0). Its profit-maximizing prices are $p_1 = p_2 = p^m(2) = V - k/4$ and its profits are $\pi^m(2) = V - k/4 - 2f$. For $3/4k > f$ the monopolist would find it profitable to introduce the second product.

    From footnote 33 on page 410 we know that costs when there is a single product equal $k/3+f$ and $k/12+2f$ when there are two products. When $k/4 > f$ the social planner would introduce the second product. For the indicated range of f, a monopolist would introduce two products, the social planner only one. The monopolist has an excessive incentive to introduce the second product since she is not concerned about the savings in transportation (disutility costs) from a better match between preferences and products. The advantage to her is that she can charge higher prices and extract more surplus with the introduction of a second product. That is, her proliferation decision is based both on the savings in transportation costs and her ability to extract a larger share of total surplus from consumers.

12. (a) The density of consumers around the circle is 200 (=population/distance). There are $100/N$ consumers in a circle segment of length $1/(2N)$ miles (density x length).

    (b) The socially optimal number of products minimizes the total cost to supply every consumer with one unit. The costs per product are: $C(N^s) = F + qN^s + T(N^s)$ where $T(N^s)$ are total transportation costs as a function of the number of firms. Cost minimization means that the products will be distributed symmetrically around the circle and thus $q = 200/N^s$ where N^s is the number of products. The transportation costs for a firm are

$$T(N^s) = 4M \int_0^{1/(2N^s)} t\, dt,$$

or

$$T(N^s) = \frac{100}{(N^s)^2}.$$

So

$$C(N^s) = F + \frac{200}{N^s} + \frac{100}{(N^s)^2}.$$

The total costs for N^s firms equals $N^s C(N^s)$ or

$$TC(N^s) = N^s F + 200 + \frac{100}{(N^s)^2}.$$

Set $dTC(N^s)/dN^s = 0$ and solve for N^s: $N^s = 5$.

(c) If $F = 0$ then each consumer would get their own product or $N^S = 200$. That is, every person would have their own personalized product that matches exactly their most preferred product. This suggests—since we do not observe it—that it is very unlikely that $F = 0$. There is a trade-off between the incentive to increase the number of firms in order better match consumers and products (and minimize transport costs) and the incentive to decrease the number of firms in order to minimize set-up costs. The optimal number trades these two off appropriately.

(d) The N firms are assume to be distributed equally. Firm *i* is in competition with firm *j* located $1/N$ away on one side and firm *h* located $1/N$ away on the other side. On the *j* side the consumer indifferent between *i* and *j* is defined by:

$$p_i + 2x = p_j + (\frac{1}{N} - x)2$$

where *x* is the distance from firm *i*. Solving for *x* the demand curve for firm *i* on this side is

$$q(p_i, p_j) = 50(p_j - p_i + \frac{2}{N}).$$

The profits of firm i are

$$\pi_i = (p_i - 1)(50(p_j - p_i + \frac{2}{N})) + (p_i - 1)(50(p_h - p_i + \frac{2}{N})),$$

since it makes sales on each side. Maximizing π_i with respect to p_i requires:

$$\frac{\partial \pi_i}{\partial p_i} = (50(p_j - p_i + \frac{2}{N})) + (50(p_h - p_i + \frac{2}{N})) - 2(p_i - 1) = 0.$$

In the symmetric Nash pricing equilibrium $p_j = p_i = p_h = p$, so this reduces to

$$\frac{4}{N} - 2(p - 1) = 0.$$

Solving,

$$p = (\frac{2}{N} + 1)$$

and each firm's sales are $200/N$ and

$$\pi(N) = \frac{400}{N^2} - F.$$

Setting $F = 4$ and setting $\pi(N) = 0$ the free-entry number of firms is 10 and the equilibrium price is 1.2.

(e) If F is sunk and we assume that zero profits means that a firm does not enter, the subgame perfect number of firms with sequential entry is $N = 5$, each 1/5 around the circle, and the price is $p = 1.4$. Incumbent's earn profits of 12, but an entrant would not enter, since the best it could do would be to capture market share of $1/10$ and its equilibrium price would be 1.2 so its profits would be zero.

Alternatively, if a firm with zero profits is assumed to enter, then the equilibrium would have 6 firms, each 1/6 around the circle, and the equilibrium price is 4/3. The next entrant would anticipate a market share of 1/12, price of 7/6 and profits of -11/9.

(f) Same as (d).

14. (a) $P^C = 0.80$ and $N^C = 1.25$.

(b)

$$\Delta CS^{BG} = \alpha\left(1.12(k)^{1/2} - P_1 - 0.32\right).$$

If $\Delta CS^{BG} > 0$ then the buyers group would accept.

(c) $\Delta PS^{SG} = \alpha(P_1 - 0.8k)$.

(d) $\Delta W^{BG+SG} = \Delta CS^{BG} + \Delta PS^{SG} = \alpha(1.12k^{1/2} - 0.8k - 0.32)$. $k^{min} = 0.15804$. If $k > k^{min}$ then $\Delta W^{BG+SG} > 0$.

(e) Profits outside of the coalition are $(1-\alpha)P^C/N^C - f = (.01)(0.80)/1.25 - 0.64 = -0.6336$. The free-entry number of firms is such that the profits of a firm outside of the coalition are 0. In the long-run the new equilibrium number of firms is $N^C = 0.125$, which since $k = 0.1$ means that only firms in the sellers group stay in the market in the long-run.

(f) $\Delta PS^{SG} = 0.99P_1$.

(g) Short Run: $\Delta W^{BG+SG} = -0.0440$.
Long Run: $\Delta W^{BG+SG} = 0.0352$.

(h) $\Delta TS = -0.0445$.